CHAPTER 6
CURRENT AND RESISTANCE

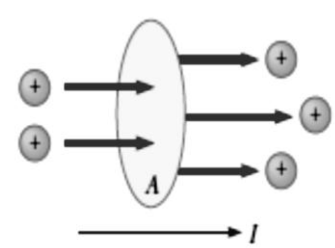
6.1 INTRODUCTION:

Thus far our treatment of electrical phenomena has been confined to the study of charges in equilibrium situations, or electrostatics. We now consider situations involving electric charges that are not in equilibrium. We use the term electric current, or simply current, to describe the rate of flow of charge through some region of space. Most practical applications of electricity deal with electric currents. For example, the battery in a flashlight produces a current in the filament of the bulb when the switch is turned on. A variety of home appliances operate on alternating current. In these common situations, current exists in a conductor, such as a copper wire. It also is possible for currents to exist outside a conductor. For instance, a beam of electrons in a television picture tube constitutes a current.

6.2 ELECTRIC CURRENT

In this section, we study the flow of electric charges through a piece of material. The amount of flow depends on the material through which the charges are passing and the potential difference across the material. Whenever there is a net flow of charge through some region, an electric current is said to exist.

It is instructive to draw an analogy between water flow and current. In many localities it is common practice to install low-flow showerheads in homes as a water conservation measure. We quantify the flow of water from these and similar devices by specifying the amount of water that emerges during a given time interval, which is often measured in liters per minute. On a grander scale, we can characterize a river current by describing the rate at which the water flows past a particular location.



To define current more precisely, suppose that charges are moving perpendicular to a surface of area A, as shown in Figure 1. (This area could be the cross-sectional area of a wire, for example.) **The current is the rate at which charge flows through this surface**. If ΔQ is the amount of charge that passes through this area in a time interval Δt, the **average current** I_{av} is equal to the charge that passes through A per unit time:

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

If the rate at which charge flows varies in time, then the current varies in time; we define the ***instantaneous current I*** as the differential limit of average current:

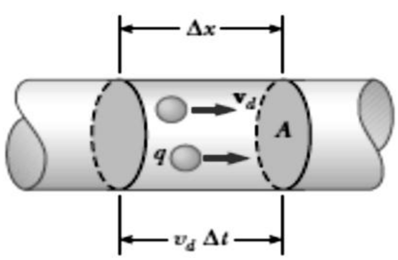
$$I = \frac{dQ}{dt} \qquad 6.1$$

Current is a scalar quantity. The SI unit of current is the ampere (A):

$$1A = \frac{1\,C}{1\,s}$$

That is, ***1 A*** of current is equivalent to ***1 C*** of charge passing through the surface area in ***1 s***.

6.3 CURRENT, DRIFT VELOCITY, AND CURRENT DENSITY



Consider there are n positive charge particle per unit volume (i.e concentration of particles; its SI unit is m^{-3}) moves in the direction of the field from the left to the right, all move in drift velocity v. In time Δt each particle moves distance $v\Delta t$ the shaded area in the figure, The volume of the shaded area in the figure is equal $Av\Delta t$, the charge ΔQ flowing across the end of the cylinder in time Δt is

ΔQ = number of carriers in section × charge per carrier = $(nA\,\Delta x)q$

where q is the charge on each carrier. If the carriers move with a speed v_d, the displacement they experience in the x direction in a time interval Δt is $\Delta x = v_d\,\Delta t$. Let us choose Δt to be the time interval required for the charges in the cylinder to move through a displacement whose magnitude is equal to the length of the cylinder. This time interval is also that required for all of the charges in the cylinder to pass through the circular area at one end. With this choice, we can write ΔQ in the form

$\Delta Q = = (nA\, v_d\, \Delta t)q$

If we divide both sides of this equation by Δt, we see that the average current in the conductor is

$$I = \frac{\Delta Q}{\Delta t} = nAqv_d \qquad 6.2$$

The speed of the charge carriers v_d is an average speed **called <u>the drift speed</u>**.

The current per unit cross-sectional area is called **the current density *J***

$$J = \frac{I}{A} = nqv_d \qquad 6.3$$

The **S.I** units of current density are amperes per square meter ***(A/m²)*. The current density is a vector quantity.**

$$\vec{J} = nq\vec{v}_d \qquad 6.4$$

There are no absolute value signs in Eq. (6.4). If q is positive, $\vec{v}_d$ is in the same direction as $\vec{E}$ if is q negative, $\vec{v}_d$ is opposite to $\vec{E}$ In either case, $\vec{J}$is in the same direction as $\vec{E}$ Equation (6.3) gives the magnitude of the vector current density .

Note that :-

Current density , $\vec{J}$ is a vector, but **current *I*** is not. The difference is that the current density , $\vec{J}$ describes how charges flow at a certain point, and the vector's direction tells you about the direction of the flow at that point. By contrast, the current ***I*** describes how charges flow through an extended object such as a wire.

Example. 1

A copper conductor of square cross section 1mm^2 on a side carries a constant current of 20A. The density of free electrons is 8×10^{28} electron per cubic meter. Find the current density and the drift velocity.

Solution

The current density is

$$J = \frac{I}{A} = 20 \times 10^6 \mathrm{A/m^2}$$

The drift velocity is

$$v_d = \frac{J}{nq} = \frac{20 \times 10^6}{(8 \times 10^{28})(1.6 \times 10^{-19})} = 1.6 \times 10^{-3} m/s$$

This drift velocity is very small compare with the velocity of propagation of current pulse, which is 3 $\times 10^8$m/s. The smaller value of the drift velocity is due to the collisions with atoms in the conductor.

6.4 RESISTANCE AND RESISTIVITY

If we apply the same potential difference between the ends of geometrically similar rods of copper and of glass, very different currents result. the ratio between the potential difference ***V*** and is the current ***I*** across the conductor. **called the electrical resistance *R***

$$R = \frac{V}{I} \qquad \textbf{(Ohm's law)} \qquad 6.5$$

This equation is known as Ohm's law, which show that a linear relationship between the potential difference and the current flowing in the conductor. Any conductor shows the lineal behavior its resistance is called *ohmic* resistance.

The resistance ***R*** has a unit of **volt/ampere (*v/A*)**, which is called ***Ohm*** **(Ω).**

$$I = \frac{V}{R} \qquad \text{and} \qquad V = RI$$

The resistance in the circuit is drown using this symbol

A ***current density J*** and an ***electric field E*** are established in a conductor whenever a potential difference is maintained across the conductor. If the potential difference is constant, then the current also is constant. In some materials, the current density is proportional to the electric field:

$$\boldsymbol{E\alpha J} \qquad \boldsymbol{\rho = \frac{E}{J}} \qquad 6.6$$

where the constant of proportionality $\boldsymbol{\rho}$ is called the **resistivity, the resistivity has unit of** $\boldsymbol{\Omega.m}$.

$$\frac{\boldsymbol{unit}\ (\boldsymbol{E})}{\boldsymbol{unit}\ (\boldsymbol{J})} = \frac{\boldsymbol{V/m}}{\boldsymbol{A/m^2}} = \frac{\boldsymbol{V}}{\boldsymbol{A}}\ \boldsymbol{m} = \boldsymbol{\Omega.m} \text{ (ohm-meters)}$$

The inverse of resistivity is known as the conductivity σ, its unit $\Omega^{-1}m^{-1}$

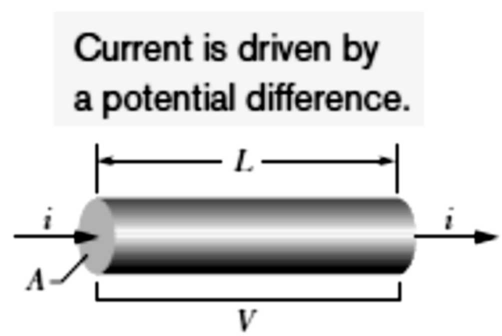
$$\boldsymbol{\sigma = \frac{1}{\rho}}$$

Nota:

- **<u>Resistance *R*</u>** is a property of an object**.**
- **<u>Resistivity ρ</u>** is a property of a material**.**

6.5 CALCULATING RESISTANCE FROM RESISTIVITY

If we know the resistivity of a substance such as copper, we can calculate the resistance of a length of wire made of that substance. Let A be the cross-sectional area of the wire, let L be its length, and let a potential difference V exist between its ends Fig. If the streamlines representing the current density are uniform throughout the wire, the electric field and the current density will be constant for all points within the wire and will have the values



$$\boldsymbol{E = \frac{V}{L}} \quad \text{and} \quad \boldsymbol{J = \frac{i}{A}} \qquad 6.7$$

We can then combine Eqs. 6.6 and 6.7 to write

$$\boldsymbol{\rho = \frac{E}{J} = \frac{V/L}{i/A}} \qquad 6.8$$

However, *V*/*i* is the resistance *R*, which allows us to recast Eq. 6.8 as

$$\boldsymbol{R = \rho\frac{L}{A}} \qquad 6.9$$

Therefore, the **<u>resistance *R*</u>** is proportional to **<u>the length *l*</u>** of the conductor and inversely proportional **<u>the cross-sectional area *A*</u>** of it

Notice that :-

<u>the resistance *R*</u> of a conductor depends on the geometry of the conductor,

the resistivity $\boldsymbol{\rho}$ of the conductor depends only on the electronic structure of the material.

Example 2

Calculate the resistance of a piece of aluminum that is 10 cm long and has a cross-sectional area of $2\times10^{-4}m^2$. What is the resistance of a piece of glass with the same dimensions? $\rho_{Al}=2.82\times10^{-8}\Omega.m$, $\rho_{glass}= 3\times10^{10}\Omega.m$.

Solution

The resistance of aluminum

$$\boldsymbol{R=\rho\frac{L}{A}=2.82\times10^{-8}\frac{(0.1)}{(2\times10^{-4})}=1.4\times10^{-4}\Omega}$$

The resistance of glass

$$\boldsymbol{R=\rho\frac{L}{A}=3\times10^{10}\frac{(0.1)}{(2\times10^{-4})}=1.5\times10^{13}\Omega}$$

The resistance of aluminum is much smaller than glass.

Example 3

A 0.90 V potential difference is maintained across a 1.5 m length of tungsten wire that has a cross-sectional area of 0.60 mm^2. What is the current in the wire? $\rho_{tungsten} = 5.6\times 10^{-8}$

Solution

From Ohm's law

$\boldsymbol{I=\frac{V}{R}}$ where $\boldsymbol{R=\rho\frac{L}{A}}$

therefore,

$$\boldsymbol{I=\frac{VA}{\rho L}=\frac{(0.90)(6\times10^{-7})}{(5.6\times10^{-8})(1.5)}=6.43A}$$

Example 4

(a) Calculate the resistance per unit length of a 22 nichrome wire of radius 0.321mm.

(b) If a potential difference of 10V is maintained cross a1m length of nichrome wire, what is the current in the wire. $\rho_{nichromes} = 1.5\times10^{-6}\ \Omega.m$.

Solution

(a) The cross-section area of the wire is

$$\mathbf{A = \pi r^2 = \pi\ (0.321\times10^{-3})^2 = 3.24\times10^{-7}m^2}$$

The resistance per unit length is R/l

$$\frac{R}{L} = \frac{\rho}{A} = \frac{(1.5\times10^{-6})}{(3.24\times10^{-7})} = 4.6\ \Omega/\text{m}$$

(b) The current in the wire is

$$I = \frac{V}{R} = \frac{10}{4.6} = 2.2A$$

Nichrome wire is often used for heating elements in electric heater, toaster and irons, since its resistance is 100 times higher than the copper wire.

6.6 RESISTANCE AND TEMPERATURE

The values of most physical properties vary with temperature, and resistivity is no exception. Figure, for example, shows the variation of this property for copper over a wide temperature range. The relation between temperature and resistivity for copper and for metals in general is fairly linear over a rather broad temperature range.

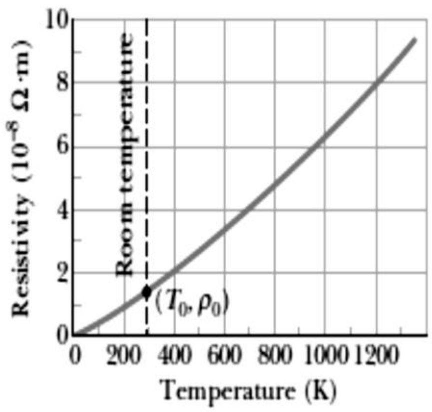
As temperature increases, the ions of the conductor vibrate with greater amplitude, making it more likely that a moving electron will collide with an ion as in Fig. This impedes the drift of electrons through the conductor and hence reduces the current.

we can write an empirical approximation that is good enough for most engineering purposes:

$$\boldsymbol{\rho - \rho_o = \rho_o \alpha (T - T_o)}$$

Here T_o is a selected reference temperature and ρ_o is the resistivity at that temperature.

The quantity α in called the ***temperature coefficient of resistivity***



Resistivity can depend on temperature.

6.7 ELECTRICAL POWER

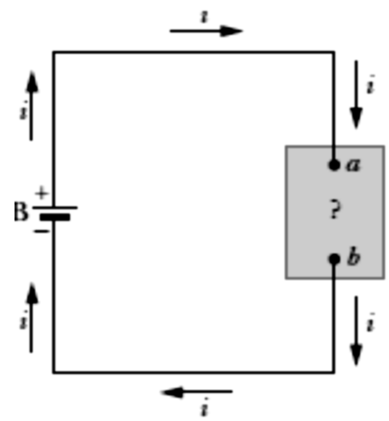


Figure shows a circuit consisting of a battery ***B*** that is connected by wires, which we assume have negligible resistance, to an unspecified conducting device. The battery maintains a potential difference of magnitude ***V*** across its own terminals and thus (because of the wires) across the terminals of the unspecified device, with a greater potential at terminal ***a*** of the device than at terminal ***b***.

Because there is an external conducting path between the two terminals of the battery, and because the potential differences set up by the battery are maintained, a steady current i is produced in the circuit, directed from terminal ***a*** to terminal ***b***. The amount of charge ***dq*** that moves between those terminals in time interval dt is equal to ***i dt***. This charge ***dq*** moves through a decrease in potential of magnitude ***V***, and thus its electric potential energy decreases in magnitude by the amount

$$\boldsymbol{dU = dqV = idtV}$$

The rate of electric energy (*dU/dt*) is an electric power (*P*).

$$\boldsymbol{\frac{dU}{dt} = iV} \qquad \text{(rate of electrical energy transfer).} \qquad 6.10$$

Moreover, this power P is also the rate at which energy is transferred from the battery to the unspecified device.

- If that device is a motor connected to a mechanical load, the energy is transferred as work done on the load.
- If the device is a storage battery that is being charged, the energy is transferred to stored chemical energy in the storage battery.
- If the device is a resistor, the energy is transferred to internal thermal energy, tending to increase the resistor's temperature

The unit of power that follows from Eq. 6-10 is the ***volt-ampere (V.A)***. We can write it as

$$\mathbf{1\,V.A = \left(1\frac{J}{C}\right)\left(1\frac{C}{s}\right) = \left(1\frac{J}{s}\right) = 1w}$$

For a resistor or some other device with resistance R, we can combine Eqs. 6.5 (***R* = *V/i***) and 6.10 to obtain, for the rate of electrical energy dissipation due to a resistance, either

$$\boldsymbol{P = i^2R} \qquad \textbf{(resistive dissipation)} \qquad \textbf{6.11}$$

Or $$\boldsymbol{P = \frac{V^2}{R}} \qquad \textbf{(resistive dissipation)} \qquad \textbf{6.12}$$

Example 5

An electric heater is constructed by applying a potential difference of 110 volt to a nichrome wire of total resistance 8 Ω. Find the current carried by the wire and the power rating of the heater.

Solution

Since ***V* = *IR***

$$\boldsymbol{I = \frac{V}{R} = \frac{110}{8} = 13.8A}$$

The **power *P*** is $$\boldsymbol{P = i^2R = (13.8)^2 \times 8 = 1520w}$$

Example 6

A light bulb is rated at 120 v/ 75*W*. The bulb is powered by a 120 v. Find the current in the bulb and its resistance.

Solution

The **Power *P*** is $$\boldsymbol{P = iV}$$

$$i = \frac{p}{V} = \frac{75}{120} = 0.625A$$

The **Resistance *R*** is

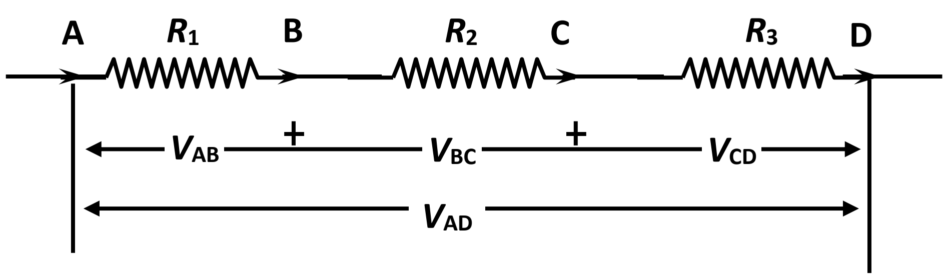
$$R = \frac{V}{i} = \frac{120}{0.625} = 192\Omega$$

6.8 COMBINATION OF RESISTORS

Some times the electric circuit consist of more than two resistors, which are, connected either in parallel or in series the equivalent resistance is evaluated as follow:

6.8.1 Resistors in Series:

Figure shown the series combination of three resistors, the currents (*I*) are the same in both resistors because the amount of charge that passes through R_1 must also pass through R_2 and R_3 respectively, in the same time interval. While the potential difference (*V*) applied across the series combination of resistors will divide between the resistors.



According to Ohm's law,

$$V = IR$$

For Combination of resistors

$$V_t = V_1 + V_2 + V_3 + ... + V_n$$

$$\therefore I_t R_t = IR_1 + IR_2 + IR_3 + ... + IR_n$$

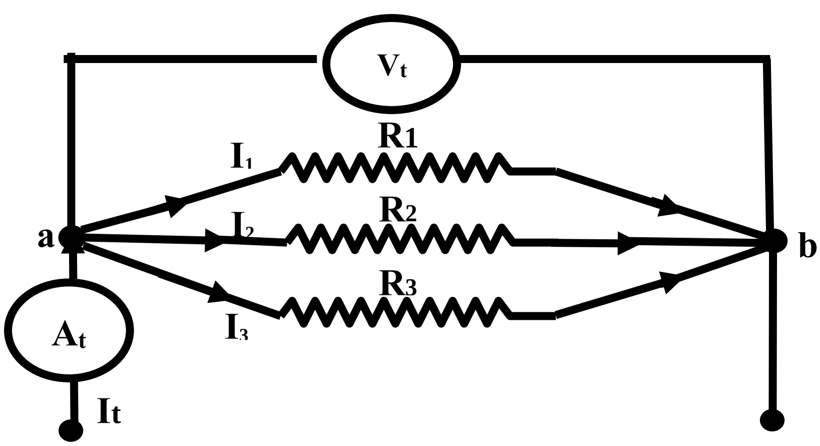
Where

$$\because I_t = I_1 = I_2 = I_3 = ...= I_n$$

$$\therefore R_t = R_1 + R_2 + R_3 + ...+ R_n$$

6.2.1 Resistors in parallel:

Figure shown the Parallel combination of three resistors, the voltage (*V*) are the same in both resistors. while the current (*I*) when charges reach point (a) called a junction, they split into three parts, with some going through R_1, R_2 and the rest going through R_3 respectively, a junction is any point in a circuit where a current can split. This split results less current in each individual resistor than the current leaving the battery. Because electric charge is conserved, the current I that enters point (a) must equal the total current at point (b).



According to Ohm's law,

$V = IR$

For Combination of resistors

$$I_t = I_1 + I_2 + I_3 + \dots\dots\dots\dots\dots + I_n$$

$$\therefore \frac{V_t}{R_t} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \dots\dots\dots\dots\dots + \frac{V_n}{R_n}$$

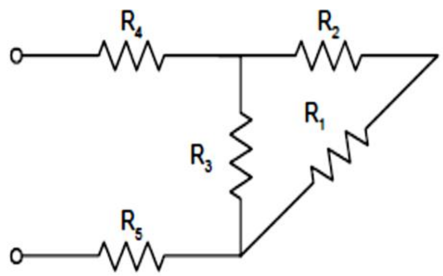
Where,

$$\because V_t = V_1 = V_2 = V_3 = \dots\dots\dots\dots\dots = V_n$$

$$\therefore \frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots\dots\dots\dots\dots + \frac{1}{R_n}$$

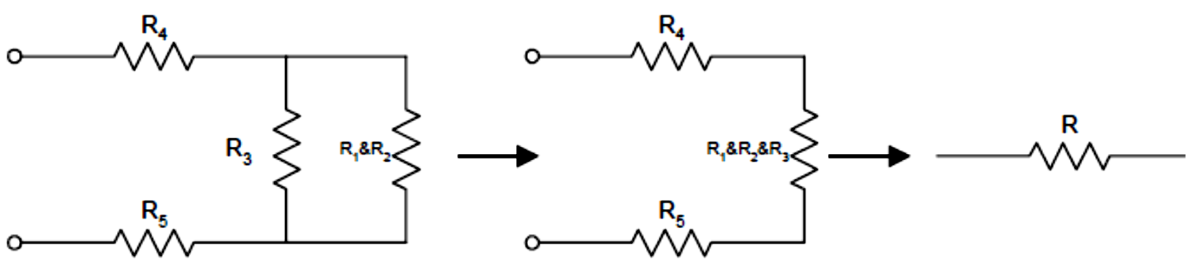
Example 7

Find the equivalent resistance for the circuit shown in figure R_1=4Ω, R_2=3Ω, R_3=3Ω, R_4=5Ω, and R_5= 2.9Ω.



Solution

Resistance $\boldsymbol{R_1}$ and $\boldsymbol{R_2}$ are connected in series therefore the circuit is simplify as shown below



$$\boldsymbol{R' = R_1 + R_2 = 4 + 3 = 7\ \Omega}$$

Then the resultant resistance of ($\boldsymbol{R'}$) are connected in parallel with resistance $\boldsymbol{R_3}$

$$\therefore \frac{1}{R''} = \frac{1}{R'} + \frac{1}{R_3} = \frac{1}{7} + \frac{1}{3} = \frac{10}{21}$$

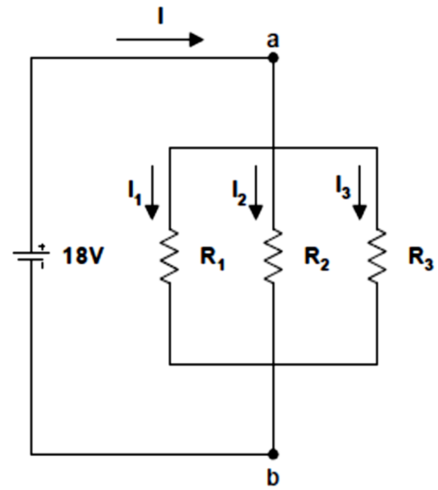
$\boldsymbol{R'' = 2.1\ \Omega}$

The resultant resistance R for R_5 & R_4 & R'' are connected in series

R= R′′+ R₅+R₄ =2.1 + 5 + 2.9 = 10Ω

Example 8

Three resistors are connected in parallel as in shown in figure. A potential difference of 18 V is maintained between points *a* and *b*.



(a) find the current in each resistor.

(b) Calculate the power dissipated by each resistor and the total power dissipated by the three resistors.

(c) Calculate the equivalent resistance, and the from this result find the total power dissipated.

Solution

To find the current in each resistors, we make use of the fact that the potential difference across each of them is equal to 18v, since they are connected in parallel with the battery.

Applying ***V=IR*** to get the current flow in each resistor and then apply ***P = I²R*** to get the power dissipated in each resistor.

$$\therefore I_1 = \frac{V}{R_1} = \frac{18}{3} = 6A \longrightarrow \therefore P_1 = I_1^2 R_1 = 108W$$

$$\therefore I_2 = \frac{V}{R_2} = \frac{18}{6} = 3A \longrightarrow \therefore P_2 = I_2^2 R_2 = 54W$$

$$\therefore I_3 = \frac{V}{R_3} = \frac{18}{9} = 2A \longrightarrow \therefore P_3 = I_3^2 R_3 = 36W$$

The equivalent resistance $\boldsymbol{R_{eq}}$ is

$$\therefore \frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{6} + \frac{1}{9} == \frac{11}{18}$$

R_{eq} = 1.6 Ω

Example 9

A 2.4 m length of wire that is 0.031cm^2 in cross section has a measured resistance of 0.24Ω. Calculate the conductivity of the material

Solution

$$R = \rho\frac{L}{A} \qquad \text{and} \quad \rho = \frac{1}{\sigma}$$

$$\sigma = \frac{L}{RA} = \frac{2.4}{(0.24)(3.1 \times 10^{-6})} = 3.23 \times 10^{6}\Omega^{-1}m^{-1}$$

Example 9

A 0.9V potential difference is maintained across a 1.5 m length of tungsten wire that has cross-sectional area of 0.6 mm^2. What is the current in the wire?

Solution

$$I = \frac{V}{R} \qquad \text{and} \qquad R = \rho\frac{L}{A}$$

$$I = \frac{VA}{\rho L} = \frac{0.9 \times 6 \times 10^{-7}}{(5.6 \times 10^{-8}) \times (1.5)} = 6.43A$$

CHAPTER 7

DIRECT CURRENT CIRCUITS

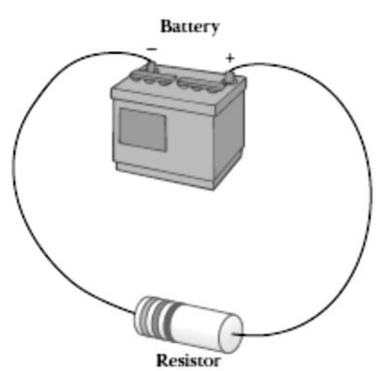
7.1 INTRODUCTION:

This chapter is concerned with the analysis of simple electric circuits that contain batteries, resistors, and capacitors in various combinations. We will see some circuits in which resistors can be combined using simple rules. The analysis of more complicated circuits is simplified using two rules **known as Kirchhoff's rules**, which follow from the laws of conservation of energy and conservation of electric charge for isolated systems. Most of the circuits analyzed are assumed to be in steady state, which means that currents in the circuit are constant in magnitude and direction.

A current that is constant in direction is called a direct current (DC). We will study alternating current (AC), in which the current changes direction periodically. Finally, we describe electrical meters for measuring current and potential difference and discuss electrical circuits in the home.

7.2 ELECTROMOTIVE FORCE

In any electrical circuit it must exist a device to provide energy to force the charge to move in the circuit, this device could be battery or generator, in general it is called electromotive force (***emf***) symbol (ξ).



The electromotive force are able to maintain a potential difference between two points to which they are connected.

The emf (ξ) of a battery: is the maximum possible voltage that the battery can provide between its terminals. You can think of a source of emf as a "charge pump." When an electric potential difference exists between two points, the source moves charges "uphill" from the lower potential to the higher.

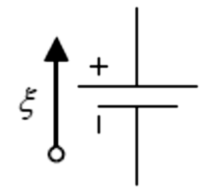
Then electromotive force (*emf*) (ξ) is defined as the work done per unit charge.

$$\because \boldsymbol{\xi} = \frac{\mathbf{dw}}{\boldsymbol{dq}} \qquad 7.1$$

The unit of ξ is *joule*/*coulomb*, which is *volt*.

The device acts as an *emf* source is drawn in the circuit as shown in the figure below, with an arrow points in the direction which the positive charge move in the external circuit. *i.e*. from the -ve terminal to the +ve terminal of the battery

When we say that the battery is 1.5 volts we mean that the emf of that battery is 1.5 volts and if we measure the potential difference across the battery we must find it equal to 1.5volt.

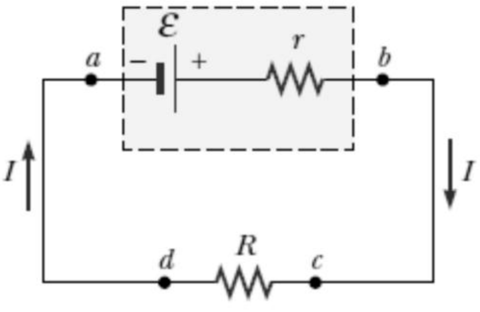


A battery provide energy through a chemical reaction, this chemical reaction transfer to an electric energy which it can be used for mechanical work Also it is possible to transfer the mechanical energy to electrical energy and the electric energy can be used to charge the battery is chemical reaction. This mean that the energy can transfer in different forms in reversible process

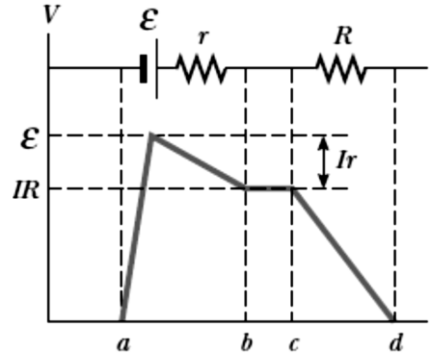
Chemical ⟶ ***Electrical*** ⟶ ***Mechanical***

7.3 FINDING THE CURRENT IN A SIMPLE CIRCUIT

- Consider the circuit shown in figure (a) where a battery is connected to a resistor ***R*** with connecting wires assuming the wires has no resistance.
- In the real situation the battery itself has some internal resistance ***r***, hence it is drawn as shown in the rectangle box in the diagram.



(a)



(b)

- Assume a +ve charge will move from point a along the loop *abcd*. In the graphical representation figure (b) it shows how the potential changes as the charge moves.
- When the charge cross the ***emf*** from point a to (b) the potential increases to *(a)* the value of ***emf*** , but when it cross the internal resistance ***r*** the potential decreases by value equal ***Ir.*** Between the point *(b)* to (*c)* the potential stay constant since the wire has no resistance. From point *(c)* to *(d)* the potential decreases by ***IR*** to the same value at point *a*.

The potential difference across the battery between point ***a*** and ***b*** is given by

$$V_b - V_a = \xi - Ir \quad 8.2$$

Note that the potential difference across points ***a*** and ***b*** is equal to the potential difference between points ***c*** and ***d*** *i.e.*

$$V_b - V_a = V_d - V_c = Ir \quad 8.3$$

Combining the equations

$$IR = \xi - Ir \quad 8.4$$

Or $$\xi = IR + Ir \quad 8.5$$

Therefore, the current ***I*** is

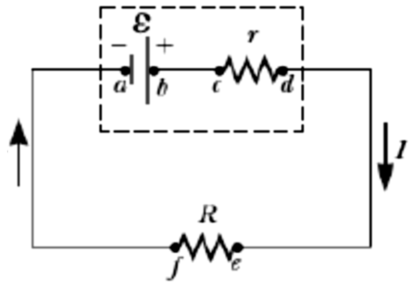
$$I = \frac{\xi}{R+r} \quad 8.6$$

This equation shows that the current in simple circuit depends on the resistors connected in series with the battery.

- ## Simple Rule

We can reach to the same answer using this rule

The algebraic sum of the changes in potential difference across each element of the circuit in a complete loop is equal to zero.



By applying the previous rule on the circuit above starting at point *a* and along the loop *abcda*

Here in the circuit we have three elements (one *emf* and two resistors *r*&*R*) applying the rule we get,

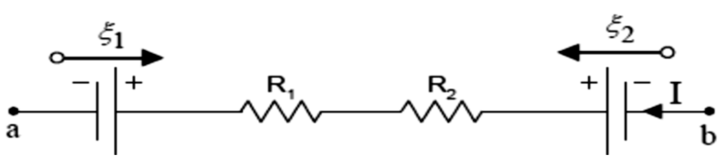
$$\xi - IR - Ir = 0 \quad 8.7$$

The +ve sine for ξ is because the change in potential from the left to the right across the battery the potential increases, the -ve sign for the change in potential across the resistors is due to the decrease of the potential as we move in the direction *abcda*.

$$I = \frac{\xi}{R+r} \quad 8.8$$

Example 1

In figure find the current flow in the branch if the potential difference V_b-V_a = 12v. Assume ζ=10v, ζ=25v, R_1=3Ω, and R_2=5Ω.



Solution

We must assume a direction of the current flow in the branch and suppose that is from point ***b*** to point ***a***.

To find the current in the branch we need to add all the algebraic changes in the potential difference for the electrical element as we move from point ***a*** to point ***b.***

$$V_b - V_a = \xi_1 + IR_1 + IR_2 - \xi_2$$

$$I = \frac{(V_b - V_a) + (\xi_2 - \xi_1)}{R_1 + R_2} = \frac{(12) + (25 - 10)}{8} = 3.37A$$

Example 2

A battery has an emf of 12.0 V and an internal resistance of 0.05 Ω. Its terminals are connected to a load resistance of 3.00 Ω.

(A) Find the current in the circuit and the terminal voltage of the battery.

(B) Calculate the power delivered to the load resistor, the power delivered to the internal resistance of the battery, and the power delivered by the battery.

Solution

a) $I = \frac{\xi}{R+r} = \frac{12V}{3.05} = 3.93A$

and the terminal voltage:

$$\Delta V = \xi - Ir = 12V - (3.93A)(0.05\Omega) = 11.8V$$

To check this result, we can calculate the voltage across the load resistance *R*:

$$\Delta V = IR = (3.93A)(3\Omega) = 11.8V$$

b) The power delivered to the load resistor is

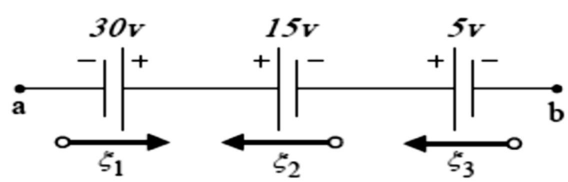
$$P_R = I^2R = (3.93A)^2(3\Omega) = 46.3W$$

The power delivered to the internal resistance is

$$P_r = I^2r = (3.93A)^2(0.05\Omega) = 0.772W$$

Example 3

Find the potential difference V_a-V_b for the branches shown in the figure.



Solution

$$V_a - V_b = +\xi_3 + \xi_2 - \xi_1 = 5 + 15 - 30 = -10V$$

This means the potential difference at point ***b*** greater then potential difference at point ***a***

7.4 KIRCHHOFF'S RULES

Simple circuits can be analyzed using the expression ΔV = IR and the rules for series and parallel combinations of resistors. Very often, however, it is not possible to reduce a circuit to a single loop.

The procedure for analyzing more complex circuits is greatly simplified if we use two principles called Kirchhoff 's rules

1. **<u>Junction rule:-</u>** The algebraic sum of the currents entering any junction must equal the sum of the currents leaving that junction.

$$\sum I_{in} = \sum I_{out} \quad 8.9$$

$$\sum I = 0 \quad \textbf{(At the junction)}$$

2. **<u>Loop rule:-</u>** The sum of the potential differences across all elements around any closed circuit loop must be zero:

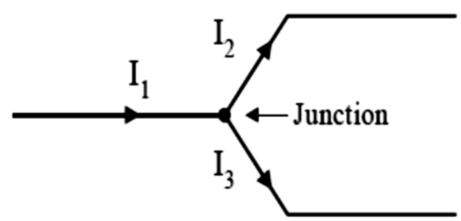
$$\sum_{\substack{closed\\loop}} \Delta V = 0 \quad \textbf{(for the loop circuit)} \quad 8.10$$

<u>Note that:-</u>

The First Kirchhoff's rule is for the current and the second for the potential difference.

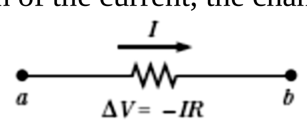
Applying the first rule on the junction shown below

$$I_1 = I_2 + I_3$$

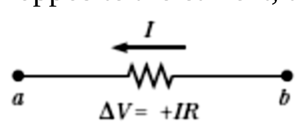


Applying the second rule on the following cases

1. If a resistor is traversed in the direction of the current, the change in potential difference across the resistor is *-IR*.

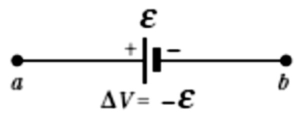


2. If a resistor is traversed in the direction opposite the current, the change in potential difference across the resistor is +*IR*.



3. If a source of *emf* is traversed in the direction of the *emf* (from (-) to (+) on the terminal), the change in potential difference is (+) .

4. If a source of *emf* is traversed in the direction opposite the *emf* (from (+) to (-) on the terminal), the change in potential difference is (-) .



Problem solving hints

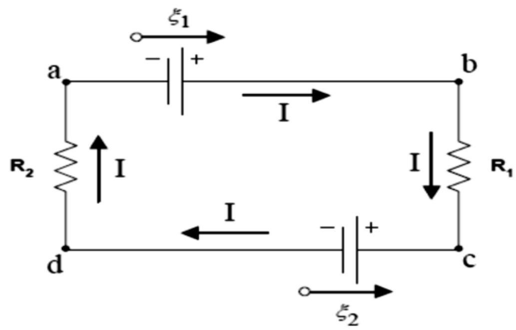
- Determine the direction of the electromotive force (***emf***) each battery in the circuit with an arrow pointing from the negative electrode to the positive electrode of the battery.
- Determine the direction of the electrical current for each component of the circuit, such as the resistance. Assuming a spcific direction for the current according to Kirchhoff's Rules
  - If the final solution shows a positive current signal, then the assumed direction is correct.
  - If the current signal appears negative, then the current value is correct, but the direction of the current is in the opposite direction to the assumed direction.
- We apply Kirchhoff's first rule to the hold in the circuit so that it is the currents signal inside to the node are positive and the outside of node is negative.
- Apply the loop rule to as many loops in the circuit as are needed to solve for the unknowns. To apply this rule, you must correctly identify the potential difference as you imagine crossing each element while traversing the closed loop (either clockwise or counter clockwise).
- Solve the equations simultaneously for the unknown quantities

Single-Loop Circuit

In a single-loop circuit there is no junctions and the current is the same in all elements of the circuit, therefore we use only the second Kirchhoff rule.

Example 4

Two battery are connected in opposite in a circuit contains two resistors as shown in figure the *emf* and the resistance are $\boldsymbol{\xi_1 = 6\ v,\ \xi_2 = 12\ v,\ R_1 = 8\ \Omega}$, ***and*** $\boldsymbol{R_2 = 10\ \Omega}$.



(a) Find the current in the circuit.

(b) What is the power dissipated in each resistor?

Solution

From figure the circuit is a single-loop circuit. We draw an arrow for each *emf* in the circuit directed from the -ve to +ve terminal of the *emf*. If we assume the current in the circuit is in the clockwise direction (*abcda*). Applying the second Kirchhoff's rule along arbitrary loop (*abcda*) we get

$$\sum_{\substack{closed \\ loop}} \Delta V = 0$$

Starting at point (*a*) to point (*b*) we find the direction of the loop is the same as the direction of the emf therefore ξ_1 is +ve, and the direction of the loop from point (*b*) to point (*c*) is the same with the direction of the current then the change in potential difference is -ve and has the value $-IR_1$. Complete the loop with the same principle as discussed before we get;

$$+\xi_1 + IR_1 - \xi_2 + IR_2 = 0$$

Solving for the current we get

$$I = \frac{+\xi_1 - \xi_2}{R_1 + R_2} = \frac{6-12}{8+10} = \frac{1}{3} A$$

The -ve sign of the current indicates that the correct direction of the current is opposite the assumed direction *i.e.* along the loop (*adba*)

The power dissipated in R_1 and R_2 is

$$P_1 = I^2 R_1 = 8/9\ \text{W}$$

$$P_2 = I^2 R_2 = 10/9\ \text{W}$$

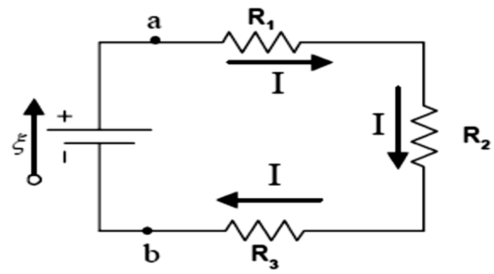
In this example the battery ξ_2 is being charged by the battery ξ_1.

Example 5

Three resistors are connected in series with battery as shown in figure, apply second Kirchhoff's rule to

(a) Find the equivalent resistance

(b) find the potential difference between the points *a* and *b*.



Solution

Applying second Kirchhoff's rule in clockwise direction we get

$$-IR_1 - IR_2 - IR_3 + \xi = 0$$

$$I = \frac{\xi}{R_1 + R_2 + R_3} = \frac{\xi}{R}$$

To find the potential difference between points a and b $V_{ab} = (V_a - V_b)$ we use the second Kirchhoff's rule along a direction starting from point (b) and finish at point (a) through the resistors. We get

$$V_a = V_b + IR$$

Where R is the equivalent resistance for R_1, R_2 and R_3

$$V_{ab} = V_a - V_b = +IR$$

The +ve sign for the answer means that $V_a > V_b$

Substitute for the current I using the equation

$$I = \frac{\xi}{R}$$

we get

$$V_{ab} = \xi$$

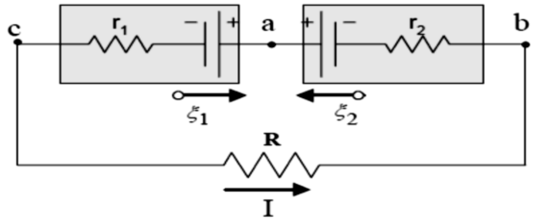
This means that the potential difference between points *a* and *b* is equal to the *emf* in the circuit (when the internal resistance of the battery is neglected).

Example 6

In the circuit shown in figure let ξ_1 and ξ_2 be 2 v and 4 v, respectively; r_1 ,r_2 and R be 1Ω, 2Ω, and 5Ω, respectively.

(a) What is the current in the circuit?

(b) What is the potential difference V_a-V_b and V_a-V_c?



Solution

Since the emf ξ_2 is larger than ξ_1 then ξ_2 will control the direction of the current in the circuit. Hence we assume the current direction is counterclockwise as shown in figure Applying the second Kirchhoff's rule in a loop clockwise starting at point *a* we get

$$-\xi_2 + Ir_2 - IR - Ir_1 + \xi_1 = 0$$

Solving the equation for the current we get

$$I = \frac{\xi_2 - \xi_1}{r_1 + r_2 + R} = \frac{4 - 2}{5 + 1 + 2} = +0.25A$$

The +ve sign for the current indicates that the current direction is correct. If we choose the opposite direction for the current we would get as a result -0.25A.

The potential difference V_a-V_b we apply second Kirchhoff's rule starting at point *b* to finishing at point *a*.

$$V_a - V_b = -Ir_2 + \xi_2 = (-0.25\times2) + 4 = +3.5V$$

Note that same result you would obtain if you apply the second Kirchhoff's rule to the other direction (the direction goes through R, r_1, and ξ_1)

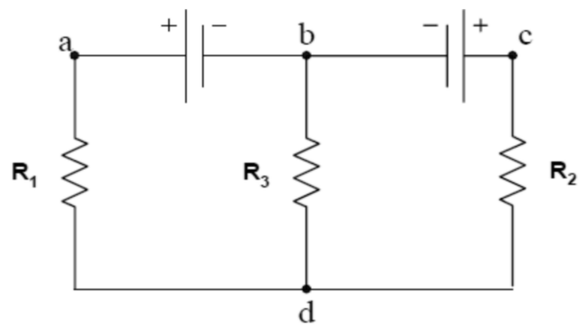
The potential difference V_a-V_c we apply second Kirchhoff's rule starting at point *c* to finishing at point *a*.

$$Va - V_b = +\xi_1 + Ir_1 = +2+(-0.25\times1) = +2.25V$$

Note that same result you would obtain if you apply the second Kirchhoff's rule to the other direction (the direction goes through R, r_2, and ξ_2)

Multi-Loop Circuit

Some circuits involving more than one current loop, such as the one shown in figure. Here we have a circuit with three loops: a left inside loop, a right inside loop, and an outside loop. There is no way to reduce this multiloop circuit into one involving a single battery and resistor.



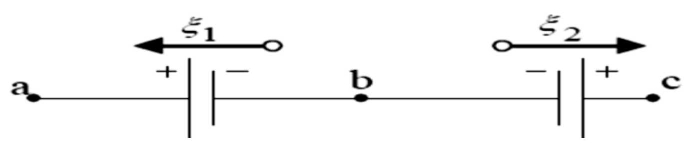
In the circuit shown above there are two junctions *b* and *d* and three branches connecting these junctions. These branches are bad, *bcd*, and *bd*.

The problem here is to find the currents in each branch.

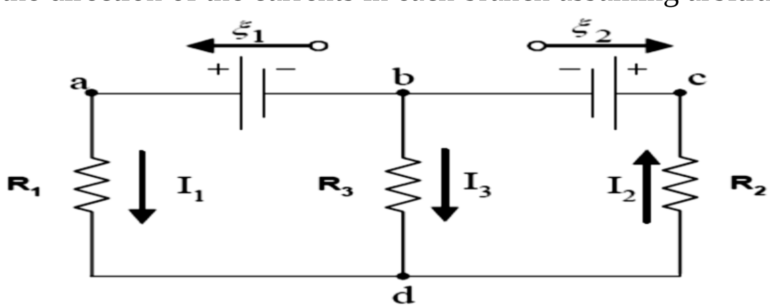
A general method for solving multi-loop circuit problem is to apply Kirchhoff's rules.

You should always follow these steps:

- Assign the direction for the *emf* from the -ve to the +ve terminal of the battery.



- Assign the direction of the currents in each branch assuming arbitrary direction.



After solving the equations the +ve sign of the current means that the assumed direction is correct, and the -ve sign for the current means that the opposite direction is the correct one.

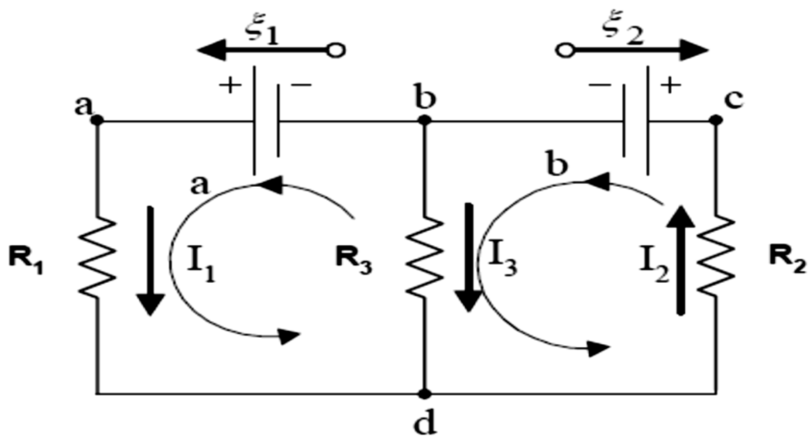
- Chose one junction to apply the first Kirchhoff's rule.

$$\sum \boldsymbol{I} = \mathbf{0}$$

At junction *d* current I_1 and I_3 is approaching the junction and I_2 leaving the junction therefore we get this equation

$$\boldsymbol{I_1 + I_3 - I_2 = 0} \qquad (1)$$

- For the three branches circuit assume there are two single-loop circuits and apply the second Kirchhoff's rule on each loop.



For loop a on the left side starting at point b we get

$$\boldsymbol{+\xi_1 - I_1R_1 + I_3R_3 = 0} \qquad (2)$$

For loop b on the left side starting at point b we get

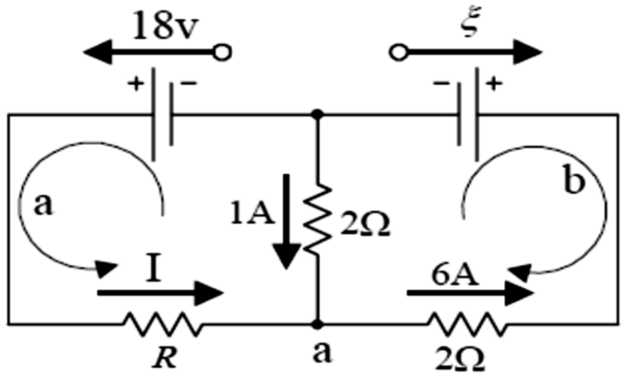
$$\boldsymbol{- I_3R_3 - I_2R_2 - \xi_2 = 0} \qquad (3)$$

Equations (1), (2), and (3) can be solved to find the unknowns currents I_1, I_2, and I_3.

The current can be either positive or negative, depending on the relative sizes of the *emf* and of the resistances.

Example 7

In the circuit shown in figure, find the unknown current *I*, resistance *R*, and *emf* ξ .



Solution

At junction a we get this equation

$$I + 1 - 6 = 0$$

Therefore the current

$$I = 5A$$

To determine *R* we apply the second Kirchhoff's rule on the loop (a), we get

$$18 - 5R + 1 \times 2 = 0 \qquad \therefore R = 4\Omega$$

To determine ξ we apply the second Kirchhoff's rule on the loop (a), we get

$$\xi + 6 \times 2 + 1 \times 2 = 0$$

$$\xi = -14V$$

Example 8

In the circuit shown in figure

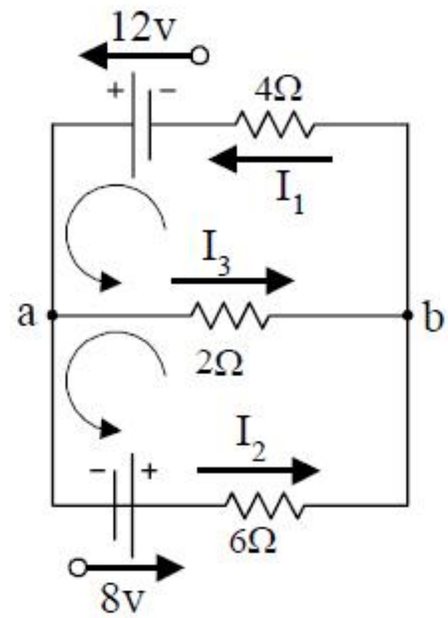
a) find the current in the 2Ω resistor,

b) the potential difference between points *a* and b

Solution

At junction *a* we get

$$I_1 = I_2 + I_3 \qquad (1)$$



For the top loop

$$12 - 2\times I_3 - 4\times I_1 = 0 \qquad (2)$$

For the bottom loop

$$8 - 6\times I_2 + 2\times I_3 = 0 \qquad (3)$$

From equation (2)

$$I_1 = 3 - 1/2\ I_3$$

From equation (2)

Substituting these values in equation (1), we get

$I_3 = 0.909A$ the current in the resistor 2Ω

The potential difference between points *a* and *b*

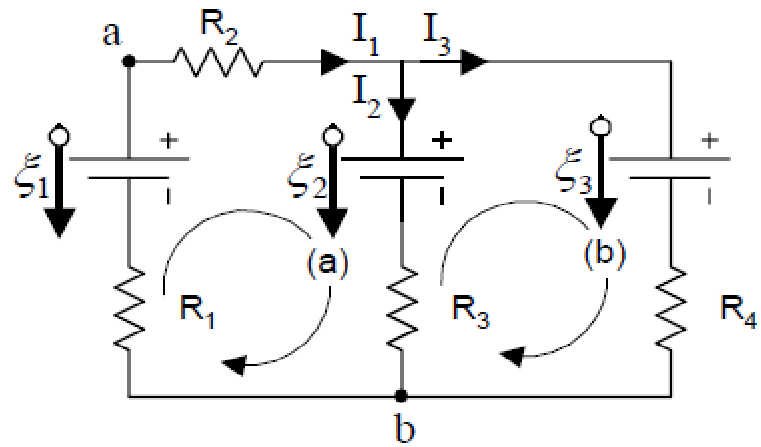
$V_a - V_b = I_3\times R = 0.909\times 2=1.82V$ $\qquad V_a > V_b$

Example 9

In the circuit shown in figure

(a) find the current I_1, I_2, and I_3

(b) the potential difference between points *a* and *b*. Use these values, ξ_1=10v, ξ_2=6v, ξ_3=4v, R_1=6Ω, R_2=2Ω, R_3=1Ω, and R_4=4Ω.



Solution

For the junction at the top we get

$$I_1 + I_2 - I_3 = 0 \qquad (1)$$

For loop a on the left side we get

$$+\xi_1 - I_1R_2 - \xi_2 + I_2R_3 - I_1R_1 = 0$$

$$+10 - 2I_1 - 6 + I_2 - 6I_1 = 0$$

$$+4 - 8I_1 + I_2 = 0 \qquad (2)$$

For loop b on the right side we get

$$- I_2R_3 + \xi_2 - \xi_2 + I_3R_4 = 0$$

$$- I_2 + 6 - 4 - 4I_3 = 0$$

$$+2 - I_2 - 4I_3 = 0 \qquad (3)$$

From equation (2)

$$I_1 = \frac{4+I_2}{8} \qquad (4)$$

From equation (3)

$$I_3 = \frac{2-I_2}{4} \qquad (5)$$

Substitute in equation (1) from equations (4)&(5) we get

$$\frac{4+I_2}{8} + I_2 - \frac{2-I_2}{4} = 0$$

$$I_2 = 0$$

From equation (4)

$I_1 = 0.5A$

From equation (4)

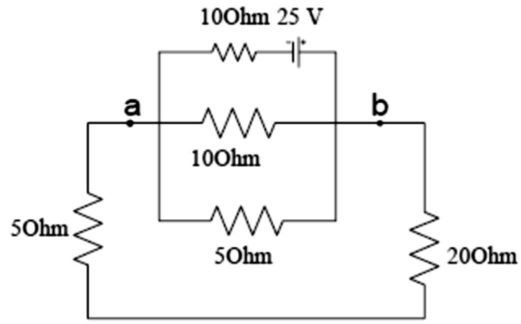
$I_3 = 0.5A$

The potential difference between points a and b we use the loop (a)

$V_b - V_a = -\xi_1 + I_1R_2$

$V_b - V_a = 10 - 0.5\times6 = -7$ V **($Vb < Va$)**

Example 10



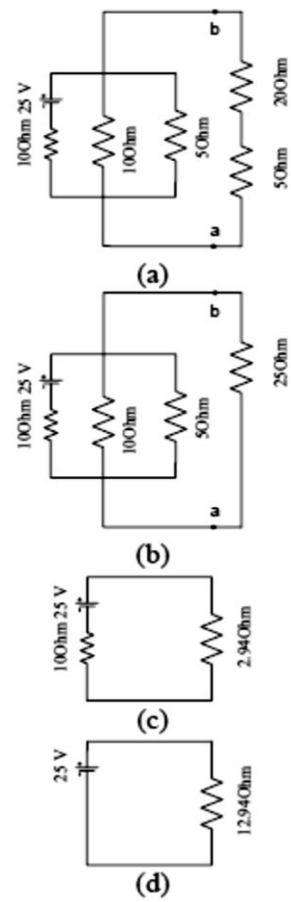
Consider the circuit shown in Figure.

Find (a) the current in the 20.0 Ω resistor

(b) the potential difference between points a and b.

Solution

Turn the diagram ion figure on its side, we find that the 20Ω and 5Ω resistors are in series, so the first reduction is as shown in (b). In addition, since the 10Ω, 5Ω, and 25Ω resistors are then in parallel, we can solve for their equivalent resistance as



$$R_{eq} = \frac{1}{\left(\frac{1}{10}+\frac{1}{5}+\frac{1}{25}\right)} = 2.95\Omega$$

This is shown in figure (c), which in turn reduces to the circuit shown in (d).

Next we work backwards through the diagrams, applying $I=V/R$ and $V=IR$. The **12.94 Ω** resistor is connected across **25 V**, so the current through the voltage source in every diagram is

$$I = \frac{V}{R} = \frac{25}{12.94} = 1.93A$$

In figure (c), the current 1.93A goes through the 2.94Ω equivalent resistor to give a voltage drop of:

$V = IR = (1.93)(2.94) = 5.68V$

From figure (b), we see that this voltage drop is the same across V_{ab}, the 10Ω resistor, and the 5Ω resistor.

Therefore,

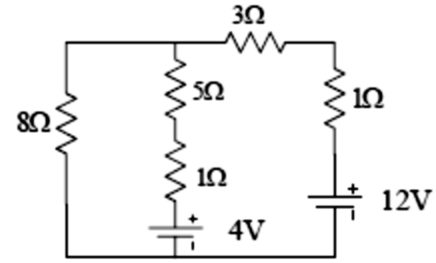
V_{ab} = 5.68V

Since the current through the 20Ω resistor is also the current through the 25Ω

$I = V_{ab}/R_{ab}$ = 5.68/25 = 0.227A

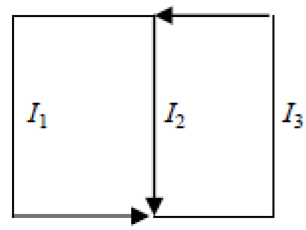
Example11

Determine the current in each of the branches of the circuit shown in figure



Solution

First we should define an arbitrary direction for the current as shown in the figure below.



$I_3 = I_1 + I_2$ **(1)**

By the voltage rule the left-hand loop

$+I_1(8Ω)-I_2(5Ω)-I_2(1Ω)-4V=0$ **(2)**

For the right-hand loop

$4V+I_2(5Ω+1Ω)+I_3(4Ω)-12V=0$ **(3)**

Substitute for I_3 from eqn. (1) into eqns. (2)&(3)

$8I_1-6I_2-4=0$ **(4)**

$4+6I_2+4(I_1+I_2)-12=0$ **(5)**

Solving eqn. (4) for I_2

$$I_2 = \frac{8I_1 - 4}{6}$$

Rearranging eqn. (5) we get

$$I_1 + \frac{10}{4}I_2 = \frac{8}{4}$$

Substitute for I_2 we get

$I_1+3.33I_1-1.67=2$

Then,

I_1=0.846 A

I_2=0.462 A

I_3=1.31 A