CHAPTER 8

MAGNETIC FIELD

8.1 INTRODUCTION:

As we have discussed, one major goal of physics is the study of how an electric field can produce an electric force on a charged object. A closely related goal is the study of how a magnetic field can produce a magnetic force on a (moving) charged particle or on a magnetic object such as a magnet. The applications of magnetic fields and magnetic forces are countless and changing rapidly every year. Here are just a few examples. For decades, the entertainment industry depended on the magnetic recording of music and images on audiotape and videotape. Although digital technology has largely replaced magnetic recording, the industry still depends on the magnets that control CD and DVD players and computer hard drives; magnets also drive the speaker cones in headphones, TVs, computers, and telephones. A modern car comes equipped with dozens of magnets because they are required in the motors for

engine ignition, automatic window control, sunroof control, and windshield wiper control. Most security alarm systems, doorbells, and automatic door latches employ magnets. In short, you are surrounded by magnets. The science of magnetic fields is physics; the application of magnetic fields is engineering. Both the science and the application begin with the question <u>"What produces a magnetic field</u>?

- Produces a magnetic field

There are two ways.

- ✓ <u>The first :-</u> Moving electrically charged particles, such as a current in a wire, to make an electromagnet. The current produces a magnetic field that can be used.
- ✓ <u>The second</u>:- produce a magnetic field is by means of elementary particles such as electrons because these particles have an intrinsic magnetic field around them.

Our first job in this chapter is to define the magnetic field B. We do so by using the experimental fact that when a charged particle moves through a magnetic field, a magnetic force F_B acts on the particle.

8.2 MAGNETIC FIELDS AND FORCES

We determined the electric field \vec{E} at a point by putting a test particle of charge q at rest at that point and measuring the electric force $\vec{F_E}$ acting on the particle. We then defined as \vec{E}

$$\vec{E} = \frac{\vec{F_E}}{a}$$

the region of space surrounding any moving electric charge also contains a magnetic field. A magnetic field also surrounds a magnetic substance making up a permanent magnet. Historically, the symbol B has been used to represent a magnetic field, and this is the notation we use in this text. The direction of the magnetic field B at any location is the direction in which a compass needle points at that location. As with the electric field, we can represent the magnetic field by means of drawings with magnetic field lines. As shown in



Figure.1. <u>Note that</u> the magnetic field lines outside the magnet point away from north poles and toward south poles. One can display magnetic field patterns of a bar magnet using small iron filings, as shown in Figure.2.





Definition of Magnetic Field B

a *magnetic field B* at some point in space in terms of the *magnetic force* F_B that the field exerts on a charged particle moving with a *velocity v*, which we call the test object.

Properties of the Magnetic Force on a charge moving in a Magnetic Field B

> The magnitude F_B of the magnetic force exerted on the particle is proportional to the charge q and to the speed v of the particle.

- > The magnitude and direction of F_B depend on the velocity of the particle and on the magnitude and direction of the magnetic field B.
- When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.
- ▶ When the particle's velocity vector makes any angle $\theta \neq 0$ with the magnetic field, the magnetic force acts in a direction perpendicular to both *v* and *B*; that is, *F*_B is perpendicular to the plane formed by *v* and *B* (Fig. 3a).
- The magnetic force exerted on a positive charge is in the direction opposite the direction of the magnetic force exerted on a negative charge moving in the same direction (Fig. 3b).
- > The magnitude of the magnetic force exerted on the moving particle is proportional to $sin \theta$, where θ is the angle the particle's velocity vector makes with the direction of *B*.



We can summarize these observations by writing the magnetic force in the form

$F_B = qvB$

8.2

We can regard this equation as definition of the magnetic field at some point in space. That is, the magnetic field is defined in terms of the force acting on a moving charged particle.

Figure 4 reviews two <u>**right-hand rules**</u> for determining the direction of the cross product $v \times B$ and determining the direction of F_B . The rule in Figure 4a depends on our right-hand rule for the cross product. Point the four fingers of your right hand along the direction of v with the palm facing B and curl them toward B. The extended thumb, which is at a right angle to the fingers, points in the direction of $v \times B$. Because $F_B = q v \times B$, F_B is in the direction of your thumb if q is positive and opposite the direction of your thumb if q is negative.





An alternative rule is shown in Figure 4b. Here the thumb points in the direction of v and the extended fingers in the direction of B. Now, the force F_B on a positive charge extends outward from your palm. The advantage of this rule is that the force on the charge is in the direction that you would push on something with your hand outward from your palm. The force on a negative charge is in the opposite direction. Feel free to use either of these two right-hand rules.

The magnitude of the magnetic force on a charged particle is

$$F_B = |q| v B \sin \theta$$

8.3

where θ is the smaller angle between v and B. From this expression, we see that F_B is zero when v is parallel or antiparallel to B ($\theta = 0$ or 180°) and maximum when v is perpendicular to B ($\theta = 90°$).

There are several important differences between electric and magnetic forces:

- The electric force acts along the direction of the electric field, whereas the magnetic force acts perpendicular to the magnetic field.
- The electric force acts on a charged particle regardless of whether the particle is moving, whereas the magnetic force acts on a charged particle only when the particle is in motion.
- The electric force does work in displacing a charged particle, whereas the magnetic force associated with a steady magnetic field does no work when a particle is displaced because the force is perpendicular to the displacement.

From the last statement and on the basis of the work–kinetic energy theorem, we conclude that the kinetic energy of a charged particle moving through a magnetic field cannot be altered by the magnetic field alone.

<u>The SI unit</u> of magnetic field is the **newton per coulomb-meter per second**, which is called the **Tesla (T):**

$$1T = 1\frac{N}{C.m/S}$$

Because a coulomb per second is defined to be an ampere, we see that

$$1T = 1\frac{N}{A.m}$$

Non-SI magnetic-field unit in common use, called the *gauss* (G), is related to the tesla through the conversion $1 \text{ T} = 10^4 \text{ G}$.

Quiz

The north-pole end of a bar magnet is held near a positively charged piece of plastic. Is the plastic (a) attracted, (b) repelled, or (c) unaffected by the magnet?

Quiz

A charged particle moves with velocity v in a magnetic field B. The magnetic force on the particle is a maximum when v is (a) parallel to B, (b) perpendicular to B, (c) zero

8.3 MAGNETIC FIELD LINES

Figure shows how the magnetic field near a bar magnet (a permanent magnet in the shape of a bar) can be represented by magnetic field lines. The lines all pass through the magnet, and they all form closed loops (even those that are not shown closed in the figure). The external magnetic effects of a bar magnet are strongest near its ends, where the field lines are most closely spaced. The (closed) field lines enter one end of a magnet and exit the other end. The end of a magnet from which the field lines emerge is called the *north pole of the magnet*; the other end, where field lines enter the magnet, is called *the south pole*. Because a magnet has two poles, it is said to **be a magnetic dipole.**



If we place two of them near each other we find:

Opposite magnetic poles attract each other, and like magnetic poles repel each other.

8.4 MAGNETIC FORCE ACTING ON A CURRENT-CARRYING CONDUCTOR

One can demonstrate the magnetic force acting on a current-carrying conductor by hanging a wire between the poles of a magnet, as shown in Figure 5a. For ease in visualization, part of the horseshoe magnet in part (a) is removed to show the end face of the south pole in parts (b), (c), and (d) of Figure 5. The magnetic field is directed into the page and covers the region within the shaded squares. *When the current in the wire is zero*, the wire remains vertical, as shown in Figure 5b. However, when the wire carries a current directed upward, as shown in Figure 5c, the wire deflects to the left. *If we reverse the current*, as shown in Figure 5d, the wire deflects to the right.



Let us this discussion by considering a straight segment of wire of length L and crosssectional area A, carrying a current I in a uniform *magnetic field B*, as shown in Figure 6. The magnetic force exerted on *a charge q* moving with *a drift velocity v_d* is $qv_d \times B$. To find the total force acting on the wire, we multiply the force $qv_d \times B$ exerted on one charge by the number of charges in the segment. Because the volume of the segment is *AL*, the number of charges in the segment is *nAL*, where *n* is the number of charges per unit volume. Hence, the total magnetic force on the wire of length *L* is

$$F_B = (qv_d \times B)nAL$$

The current in the wire is $I = nqv_dA$. Therefore,

$$F_B = IL \times B$$

Fig.6 Fig.6 \mathbf{v}_d \mathbf{v}_d \mathbf{v}_d \mathbf{v}_d

where *L* is a vector that points in the direction of the *current I* and has a magnitude equal to the *length L* of the segment. in *a uniform magnetic field*.

8.4

8.5 TORQUE ON A CURRENT LOOP

Figure *a* shows a rectangular loop of sides *a* and *b*, carrying *current i* through uniform *magnetic field B*. The loop in the field is placed in a *magnetic field* so that the normal \hat{n} to the loop forms an angle θ with \vec{B} . The magnitude of the *magnetic force on sides* 1 and 3 is

 $F_1 - F_3 = iaB\sin 90 = iaB$

Also, the *magnetic force on sides* 2 and 4 is

 $F_2 - F_4 = iaB\sin(90 - \theta) = iaB\cos\theta$

You can show that the force acting on side F_4 has the same magnitude as F_2 but the opposite direction. Thus, F_4 and F_2 cancel out exactly. Their net force is zero and, because their common line of action is through the center of the loop, their net torque is also zero.

The moment arm for F_1 and F_2 is equal (b/2) sin θ . The two torques tend up rotate the loop in the same (clockwise) direction and thus add up.

The net toque

 $\tau = \tau_1 - \tau_3 = (iabB/2)\sin\theta + (iabB/2)\sin\theta = iaB\sin\theta = iAB\sin\theta$

8.6 THE MAGNETIC DIPOLE MOMENT

The torque of **a** *coil* that has *N* loops, or *turns* exerted by a uniform magnetic field and carries a current *i* is given by Eq. $\tau = NiAB$. We define a new Vector μ associated with

the coil which is know as the *Magnetic dipole moment of* the coil The magnitude *Magnetic dipole moment* μ is given by $\mu = NiA$. in which *N* is the number of turns in the coil, *i* is the current through the coil, and *A* is the area enclosed by each turn of the coil. the unit of μ is the ampere–square meter (A .m²) or joule per tesla (J/T) and its direction is perpendicular to the plan of the coil The torque on the coil due to a magnetic field can be written by.

$\tau = \mu B sin \theta$

in which θ is the angle between the vectors $\boldsymbol{\tau}$ and $\boldsymbol{\mu}$





The magnetic moment vector attempts to align with the magnetic field.



We can generalize this to the vector relation

$\vec{\tau} = \vec{\mu} \times \vec{B}$

The potential energy of the coil is $U = -\mu B \cos \theta = -\vec{\mu} \times \vec{B}$ *U* has a minimum value of $-\mu B$ for $\theta = 0$ (stable equilibrium). *U* has a maximum value of μB for $\theta = 180^{\circ}$ (unstable equilibrium). *Note: For both positions the net torque is* $\tau = 0$.

8.7 THE HALL EFFECT

In 1879 Edwin Hall carried out an experiment in which he was able to determine that conduction in metals is due to the motion of **negative** charges (electrons). He was also able to determine the concentration **n** of the electrons. He used a strip of copper of width **d** and thickness ℓ . He passed a current **i** along the length of the strip and applied a magnetic field **B** perpendicular to the strip as shown in the figure. In the presence of \vec{B} the electrons experience a magnetic force \vec{F}_B that pushes them to the right (labeled R) side of the strip. This accumulates negative charge on the **R**-side and leaves the left side (labeled L) of the strip positively charged. As a result of the accumulated charge, an electric field \vec{E} is generated as shown in the figure, so that the electric force balances the magnetic force on moving charges:

 $F_{E} = F_{B}$ $eE = ev_{d}B$ $E = v_{d}B$ $\therefore J = nev_{d}$ $\therefore v_{d} = \frac{J}{ne} = \frac{i}{Ane} = \frac{i}{\ell dne}$ 8.6

Hall measured the potential difference V between the left and the right side of the metal strip:

8.7

We substitute *E* from eq. 8.7 and v_d from eq. 8.6 into eq. 8.5 and get:

$$\frac{V}{d} = B \frac{i}{\ell dne}$$



8.8

$$n=rac{Bi}{V\ell e}$$

Figs. *a* and *b* were drawn assuming that the carriers are electrons. In this case if we define $V=V_L-V_R$ we get a *positive* value.

If we assume that the current is due to the motion of positive charges (see fig. c) then positive charges accumulate on the R-side and negative charges on the L-side,

Thus, $V = V_L - V_R$ is now a *negative* number.

By determining the polarity of the voltage that develops between the left–and right–hand sides of the strip, Hall was able to prove that current was composed of moving electrons. From the value of V using equation 8.8 he was able to determine the concentration of the negative charge carriers.

8.8 THE BIOT–SAVART LAW

Shortly after Oersted's discovery in 1819 that a compass needle is deflected by a current-carrying conductor, Jean-Baptiste Biot (1774–1862) and Félix Savart (1791–1841)

performed quantitative experiments on the force exerted by an electric current on a nearby magnet. From their experimental results, Biot and Savart arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the field. That expression is based on the following experimental observations for the magnetic field dB at a point P associated with a length element ds of a wire carrying a steady current I (Fig. 7):



- The vector $d\mathbf{B}$ is perpendicular both to $d\mathbf{s}$ (which points in the direction of the current) and to the unit vector r^{2} directed from $d\mathbf{s}$ toward P.
- The magnitude of *dB* is inversely proportional to *r*², where *r* is the distance from *ds* to *P*.
- The magnitude of *dB* is proportional to the current and to the magnitude *ds* of the length element *ds*.
- The magnitude of *dB* is proportional to *sin θ*, where *θ* is the angle between the vectors *ds* and *r*².

These observations are summarized in the mathematical expression known today as the Biot–Savart law:

$$dB = \frac{\mu_o}{4\pi} \frac{Ids \times \hat{r}}{r^2}$$
8.9

where μ_o is a constant called the permeability of free space: $\mu_o = 4 \times 10^{-7} T.m/A$

Note that

The field dB in Equation 8.9 is the field created by the current in only a small length element ds of the conductor. To find the total magnetic field B created at some point by a current of finite size, we must sum up contributions from all current elements I ds that make up the current.

That is, we must evaluate B by integrating Equation 8.9

$$dB = \frac{\mu_o I}{4\pi} \int \frac{ds \times \hat{r}}{r^2}$$
8.10

where the integral is taken over the entire current distribution. This expression must be handled with special care because the integrand is a cross product and therefore a vector quantity.

✓ Magnetic Field Due to a Current in a Long Straight Wire

The magnitude of the magnetic field at a perpendicular distance R from a long (infinite) straight wire carrying a current i is given by

$$dB = \frac{\mu_o I}{4\pi R}$$
 8.11

The field magnitude B in Eq. 8.11 depends only on the current and the perpendicular distance R of the point from the wire. We shall show in our derivation that the field lines B of form concentric circles around the wire, as Fig. 8 shows. The increase in the spacing of the lines in Fig. 8 with increasing distance from the wire represents the 1/R decrease in the magnitude of predicted by Eq. 8.11. The lengths of the two vectors in the figure also show the 1/R decrease.



Here is a simple right-hand rule for finding the direction of the magnetic field set up by a current-length element, such as a section of a long wire:



Right-hand rule:

Grasp the element in your right hand with your extended thumb pointing in the direction of the current. Your fingers will then naturally curl around in the direction of the magnetic field lines due to that element.

8.9 AMPÈRE'S LAW

Oersted's 1819 discovery about deflected compass needles demonstrates that a currentcarrying conductor produces a magnetic field. Figure 9a shows how this effect can be demonstrated in the classroom. Several compass needles are placed in a horizontal plane near a long vertical wire. When no current is present in the wire, all the needles point in the same direction (that of the Earth's magnetic field), as expected. When the wire carries a strong, steady current, the needles all deflect in a direction tangent to the circle, as in Figure 9b.



We find that **B** is proportional to the **current** *i* and inversely proportional to the **distance R** from the wire.

Now let us evaluate the product *B.ds* for a small length element *ds* on the circular path defined by the compass needles, and sum the products for all elements over the closed circular path.

Along this path, the vectors ds and B are parallel at each point (see Fig. 9b), so B. ds = B ds. Furthermore, the magnitude of B is constant on this circle and is given by Equation 8.11. Therefore, the sum of the products Bds over the closed path, which is equivalent to the line integral of B.ds, is

$$\oint Bds = B \oint ds = \frac{\mu_o I}{2\pi r} 2\pi r = \mu_o I$$

where $\oint ds = 2\pi r$ is the circumference of the circular path. Although this result was calculated for the special case of a circular path surrounding a wire, it holds for a closed path of *any* shape (an *amperian loop*) surrounding a current that exists in an unbroken circuit. The general case, known as Ampère's law, can be stated as follows:

The line integral of *B.ds* around any closed path equals $\mu_o I$, where *I* is the total steady current passing through any surface bounded by the closed path.

$$\oint Bds = \mu_0 I$$
8.12

Ampère's law describes the creation of magnetic fields by all continuous current configurations, but at our mathematical level it is useful only for calculating the magnetic field of current configurations having a high degree of symmetry. Its use is similar to that of Gauss's law in calculating electric fields for highly symmetric charge distributions.



Then we use the following **curled**—**straight right-hand rule** to assign a plus sign or a minus sign to each of the currents.

Curl your right hand around the Amperian loop, with the fingers pointing in the direction of integration. A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign.

8.10 CLASSIFICATION OF MAGNETIC SUBSTANCES

Substances can be classified as belonging to one of three categories, depending on their magnetic properties.

- ✓ Paramagnetic and ferromagnetic materials are those made of atoms that have permanent magnetic moments.
- ✓ Diamagnetic materials are those made of atoms that do not have permanent magnetic moments.

For paramagnetic and diamagnetic substances, the magnetization vector M is proportional to the magnetic field strength H. For these substances placed in an external magnetic field, we can write

$$M = \chi H$$
 8.13

where χ (Greek letter chi) is a dimensionless factor called the magnetic susceptibility. It can be considered a measure of how *susceptible* a material is to being magnetized.

For paramagnetic substances

- χ is positive and *M* is in the same direction as *H*.

For diamagnetic substances

- χ is negative and **M** is opposite *H*.

the magnetic flux density or the magnetic induction is.

$B = \mu_m H$

where the constant μ_m is called the magnetic permeability of the substance and is related to the susceptibility by

$$\mu_m = \mu_o (1 - \chi) \tag{8.15}$$

Substances may be classified in terms of how their magnetic permeability μ_m compares with μ_o (the permeability of free space), as follows:

Paramagnetic $\mu_m > \mu_o$

Diamagnetic $\mu_m < \mu_o$

Because χ is very small for paramagnetic and diamagnetic substances (see Table), μ_m is nearly equal to μ_o for these substances. For ferromagnetic substances, however, μ_m is typically several thousand times greater than μ_o (meaning that χ is very large for ferromagnetic substances).

Paramagnetic Substance	x	Diamagnetic Substance	x
Aluminum	2.3×10^{-5}	Bismuth	-1.66×10^{-5}
Calcium	$1.9 imes 10^{-5}$	Copper	-9.8×10^{-6}
Chromium	2.7×10^{-4}	Diamond	-2.2×10^{-5}
Lithium	$2.1 imes 10^{-5}$	Gold	$-3.6 imes 10^{-5}$
Magnesium	$1.2 imes 10^{-5}$	Lead	-1.7×10^{-5}
Niobium	$2.6 imes 10^{-4}$	Mercury	-2.9×10^{-5}
Oxygen	2.1×10^{-6}	Nitrogen	-5.0×10^{-9}
Platinum	$2.9 imes 10^{-4}$	Silver	-2.6×10^{-5}
Tungsten	6.8×10^{-5}	Silicon	-4.2×10^{-6}

8.14

1. FERROMAGNETISM

A small number of crystalline substances exhibit strong magnetic effects called ferromagnetism. Some examples of ferromagnetic substances are iron, cobalt, nickel, gadolinium, and dysprosium. These substances contain permanent atomic magnetic moments that tend to align parallel to each other even in a weak external magnetic field. Once the moments are aligned, the substance remains magnetized after the external field is removed. This permanent alignment is due to a strong coupling between neighboring moments, a coupling that can be understood only in quantum mechanical terms.

All ferromagnetic materials are made up of microscopic regions called domains, regions within which all magnetic moments are aligned. These domains have volumes of about 10^{-12} to 10^{-8} m³ and contain 10^{17} to 10^{21} atoms. The boundaries between the various domains having different orientations are called domain walls. In an unmagnetized sample, the magnetic moments in the domains are randomly oriented so that the net magnetic moment is zero, as in Figure 10a. When the sample is placed in an external magnetic field \mathbf{B}_0 , the size of those domains with magnetic moments aligned with the field grows, which results in a magnetized sample, as in Figure 10b. As the external field becomes very strong, as in Figure 10c, the domains in which the magnetic moments are not aligned with the field become very small. When the external field is removed, the sample may retain a net magnetization in the direction of the original field. At ordinary temperatures, thermal agitation is not sufficient to disrupt this preferred orientation of magnetic moments



Fig.10

2. PARAMAGNETISM

Paramagnetic substances have a small but positive magnetic susceptibility ($0 < \chi << 1$) resulting from the presence of atoms (or ions) that have permanent magnetic moments. These moments interact only weakly with each other and are randomly oriented in the absence of an external magnetic field. When a paramagnetic substance is placed in an external magnetic field, its atomic moments tend to line up with the field. However, this alignment process must compete with thermal motion, which tends to randomize the magnetic moment orientations.

Pierre Curie (1859–1906) and others since him have found experimentally that, under a wide range of conditions, the magnetization of a paramagnetic substance is proportional to the applied magnetic field and inversely proportional to the absolute temperature:

$$M = G \frac{B_o}{T}$$
8.16

This relationship is known as Curie's law after its discoverer, and the constant *C* is called Curie's constant. The law shows that when $B_0=0$, the magnetization is zero, corresponding to a random orientation of magnetic moments.

3. DIAMAGNETISM

When an external magnetic field is applied to a diamagnetic substance, a weak magnetic moment is induced in the direction opposite the applied field. This causes diamagnetic substances to be weakly repelled by a magnet. Although diamagnetism is present in all matter, its effects are much smaller than those of paramagnetism or ferromagnetism, and are evident only when those other effects do not exist.

Diamagnetic materials are repelled by a magnetic field; an applied magnetic field creates an induced magnetic field in them in the opposite direction, causing a repulsive force. In contrast, paramagnetic and ferromagnetic materials are attracted by a magnetic field. Diamagnetism is a quantum mechanical effect that occurs in all materials; when it is the only contribution to the magnetism, the material is called diamagnetic. In paramagnetic and ferromagnetic substances, the weak diamagnetic force is overcome by the attractive force of magnetic dipoles in the material. The magnetic permeability of diamagnetic materials is less than μ 0, the permeability of vacuum. In most materials, diamagnetism is a weak effect which can only be detected by sensitive laboratory instruments, but a superconductor acts as a strong diamagnet because it repels a magnetic field entirely from its interior.

8.11 FARADAY'S LAW OF INDUCTION

To see how an emf can be induced by a changing magnetic field, consider a loop of wire connected to a sensitive ammeter, as illustrated in Figure 11. When a magnet is moved toward the loop, the galvanometer needle deflects in one direction, arbitrarily shown to the right in Figure 11a. When the magnet is brought to rest and held

stationary relative to the loop (Fig. 11b), no deflection is observed. When the magnet is moved away from the loop, the needle deflects in the opposite direction, as shown in Figure 11c. Finally, if the magnet is held stationary and the loop is moved either toward or away from it, the needle deflects. From these observations, we conclude that the loop detects that the magnet is moving relative to it and we relate this detection to a change in magnetic field. Thus, it seems that a relationship exists between current and changing magnetic field.

These results are quite remarkable in view of the fact that a current is set up even though no batteries are present in the circuit! We call such a current an *induced current* and say that it is produced by an *induced emf*.



Fig.11

Now let us describe an experiment conducted by Faraday and illustrated in Figure 12. A primary coil is connected to a switch and a battery. The coil is wrapped around an iron ring, and a current in the coil produces a magnetic field when the switch is closed. A secondary coil also is wrapped around the ring and is connected to a sensitive ammeter. No battery is present in the secondary circuit, and the secondary coil is not electrically connected to the primary coil. Any current detected in the secondary circuit must be induced by some external agent. Initially, you might guess that no current is ever detected in the secondary circuit. However, something quite amazing happens when the switch in the primary circuit is either opened or thrown closed. At the instant the switch is closed, the galvanometer needle deflects in one direction and then returns to zero. At the instant the switch is opened, the needle deflects in the opposite direction and again returns to zero.

Finally, the galvanometer reads zero when there is either a steady current or no current in the primary circuit. The key to understanding what happens in this experiment is to note first that when the switch is closed, the current in the primary circuit produces a magnetic field that penetrates the secondary circuit. Furthermore, when the switch is closed, the magnetic field produced by the current in the primary circuit changes from zero to some value over some finite time, and this changing field induces a current in the secondary circuit.



Fig.12

As a result of these observations, Faraday concluded that an electric current can be induced in a circuit (the secondary circuit in our setup) by a changing magnetic field. The induced current exists for only a short time while the magnetic field through the secondary coil is changing. Once the magnetic field reaches a steady

value, the current in the secondary coil disappears. In effect, the secondary circuit behaves as though a source of emf were connected to it for a short time. It is customary to say that an induced emf is produced in the secondary circuit by the changing magnetic field.

The experiments shown in Figures 11 and 12 have one thing in common: in each case, an emf is induced in the circuit when the magnetic flux through the circuit changes with time. In general,

"The emf induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit."

This statement, known as Faraday's law of induction, can be written

$$\varepsilon = -\frac{d\phi_B}{dt}$$
8.17

where $\phi_B = \int B dA$ is the magnetic flux If the circuit is a coil consisting of *N* loops all of the same area and if ϕ_B is the magnetic flux through one loop, an emf is induced in every loop. The loops are in series, so their emfs add; thus, the total induced emf in the coil is given by the expression

$$\varepsilon = -N\frac{d\phi_B}{dt}$$
8.18

Suppose that a loop enclosing an area *A* lies in a uniform magnetic field B, as in Figure 13. The magnetic flux through the loop is equal to $BA \cos\theta$; hence, the induced emf can be expressed as

$$\varepsilon = -\frac{d}{dt} (BA\cos\theta)$$
 8.19

From this expression, we see that an emf can be induced in the circuit in several ways:

- \checkmark The magnitude of B can change with time.
- \checkmark The area enclosed by the loop can change with time.
- The angle θ between B and the normal to the loop can change with time.
- \checkmark Any combination of the above can occur.

